УДК 514.18

MONITORING OF CHANGES IN GROUNDWATER VOLUME USING SIMPLEX WEIGHT INTERPOLATION

Sydorenko Iu.V., Ph.D.,
suliko3@ukr.net, ORCID: 0000-0002-1953-0410
Tarnavski Yu.A., Ph.D.,
tarnavski.yu@gmail.com, ORCID: 0000-0003-3226-3107
Onysko A.I., Ph.D.,
kw_fedun@ukr.net, ORCID: 0000-0001-7178-1471
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"(Kyiv, Ukraine)

The article considers the representation of the object in a vector field for calculating the change in the volume of groundwater when determining the concentration of pollutants. Simplex weighted interpolation methods are used in the modeling process.

Ever since mankind began to use nuclear power plants, it became necessary to check the environment for radioactive contamination. This task became especially relevant during the accident at the Chernobyl nuclear power plant. In our time, humanity is facing serious challenges in energy, which are connected both with natural disasters, such as floods, tsunamis, earthquakes, and with the results of enemy shelling during a full-scale war, which was unleashed by Russia. Many energy infrastructure facilities have been destroyed by the enemy, and the heavy load has now fallen on nuclear plants. Therefore, the relevance of the task of environmental monitoring takes on a new meaning.

During the inspection of the contamination of the aquifer in the area of the "Shelter" facility of the Chernobyl nuclear power plant, the task of tracking changes in radioactive contamination, taking into account the annual amplitude of groundwater fluctuations, arose. As it turned out, the concentration of radioactive elements in water changes when the groundwater level changes. This is due to a decrease in the volume of groundwater, and not as a manifestation of an increase in the number of pollutants. Therefore, there is a need to monitor the change in the total volume of water in the aquifer on the one hand, and its effect on the concentration of radioactive elements on the other.

From a large number of methods that can be used to solve this problem, deformation modeling methods were chosen by representing the object in a vector field using simplex weighted interpolation [4].

The purpose of the research is to improve the method of determining the change in the volume of the aquifer based on the georesearch database, taking into account the current situation of groundwater. The article proposes the use of simplex weighted interpolation to construct a vector field to obtain a suitable model and the possibility of visual tracking of the process of changing the groundwater

level in real time.

Keywords: deformation modeling, vector field, simplex, simplex weighted interpolation.

Problems staging. In the previous articles, the method of constructing a two-dimensional simplicial complex was given and the mathematical apparatus of the generalized weighted interpolation function was described for conducting research in the field of deformation modeling of geometric objects. The results of these studies should be used to visualize the change in the concentration of pollutants when the volume of the groundwater layer changes.

In order to carry out research, it is necessary to have geospatial data and information about the current state of groundwater near the nuclear plant.

Analysis of recent research. The simplex interpolation proposed in [1] makes it possible to obtain a continuous function of a predetermined order of smoothness in the region formed by a convex polyhedron with vertices at the points of the interpolation grid. But outside this area, the function remains undefined. Works [2-3] describe a smooth and continuous function, which is defined outside the above-mentioned region and, in combination with a simplex weighted interpolant, is continuous and smooth on the boundary of the interpolation region. The algorithm for constructing such a function is also given. In [4], it was proposed to use a vector field of deformation instead of transformations of the geometric basis for polypoint transformations. This greatly simplifies the process of modeling deformation processes. The construction of curved lines using the Gauss interpolation parametric function is described in [5]. This method allows you to interpolate closed geometric objects. The properties of the Gaussian function are affected by the variable coefficient alpha. The results of studies of the variable coefficient are given in [6].

Goals of the article. The purpose of the study is to model the deformation of geometric objects in a vector field constructed by simplex weighted interpolation to determine the change in the volume of the aquifer based on the georesearch database, taking into account the current situation of groundwater.

Main part. Simplex interpolation defines a function in simplexes, which allows you to interpolate it on regions of arbitrary shape.

Let an interpolation frame with points (xi,yi) be given, where xi, yi are arbitrary numbers, i=1..n, where n is the number of frame points. Each point is assigned a zi value. We need to find a function F(x,y) that is defined and continuous in the region formed by a convex polyhedron with vertices at the points (xi, yi), and such that F(xi, yi)=zi.

On any arbitrary set of four or more points on R2, a two-dimensional simplicial complex can be constructed that will completely cover a convex polygon with vertices at the points of the set, as illustrated in Figure 1. We define such a complex as a set of simplexes Sxijk, in which each simplex has the points (xi, yi), (xj, yj) and (xk, yk) as its vertices.

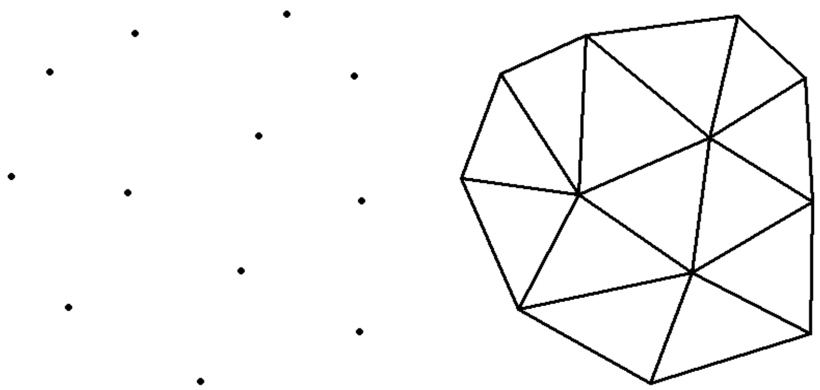

Fig. 1. A simple complex built on a point frame

For each point, we will construct a reference function fi(x,y), which will be defined and continuous in all simplexes that include the point (xi, yi), and for which fi(xi, yi)=zi will be fulfilled. You can use both biquadratic, bilinear, constant support functions [1], and any functions that are defined and continuous over the entire region of space that belongs to the polyhedron.

In each Sxijk simplex, we define the generalized weighted interpolation function as follows:

$$F_{ijk}(x,y) = \frac{f_{ij}(x,y)w(h^{ij}) + f_{jk}(x,y)w(h^{jk}) + f_{ki}(x,y)w(h^{ki})}{w(h^{ij}) + w(h^{jk}) + w(h^{ki})}, \qquad (1)$$

where each of the support functions has the form:

$$f_{ij}(x,y) = \frac{f_i(x,y)q(h_i^{ij}) + f_j(x,y)q(h_j^{ij})}{q(h_i^{ij}) + q(h_j^{ij})}.$$

Coefficients w(h), q(h) are arbitrary functions satisfying the condition:

$$\lim_{h \to 0+} (w(h)) = \infty;$$
$$\lim_{h \to 0+} (q(h)) = \infty,$$

defined, continuous and continuously decreasing at h=(0,∞);

hij is the distance from the point (x,y) to the segment formed by the points (xi, yi), (xj, yj);

h_i^{ij} – is the distance from the projection of the point (x,y) to the segment formed by the points (xi, yi), (xj, yj) to the point (xi, yi).

Figure 2 shows a separate simplex: the point at which the calculation is carried out; distances from the point to the borders of the simplex; distances from the projection of the point onto the border to the corresponding vertices.

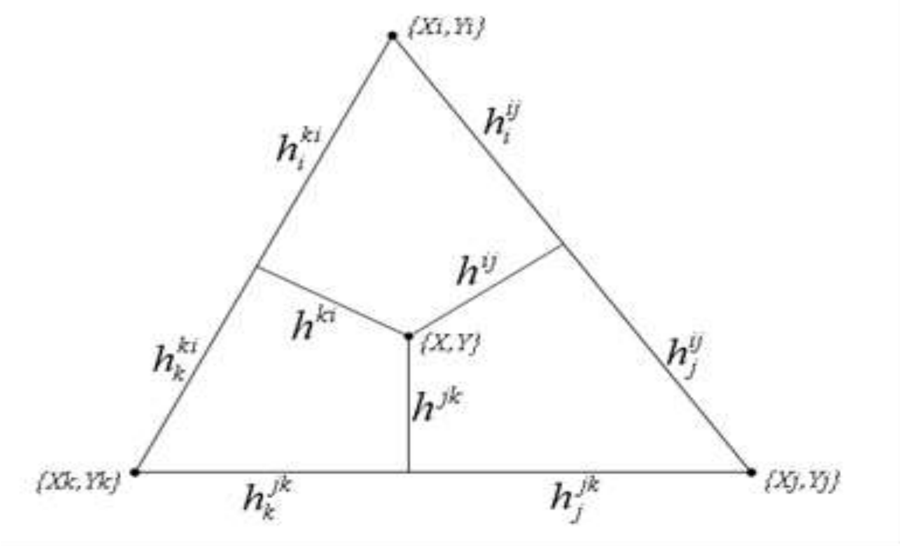

Fig. 2. Simplex S_{ijk}

Due to the conditions of definition and continuity of w(h), q(h), and fi(x, y) on the domain of the simplex, it is possible to assert the definition and continuity of the interpolant [2], but only in the domain, because on the faces of the simplex w(h) may be uncertain. It can be asserted that when approaching a certain face ij of the simplex Sijk, the following is true:

$$F_{ijk}(x, y) \rightarrow f_{ij}(xy).$$

If we supplement the definition of the interpolant at hij=0 as follows:

$$F_{ijk}(x,y) = f_{ij}(xy), \tag{2}$$

then the continuity conditions on the edges will be fulfilled for it.

Similarly, fij(x) is not defined at the vertices of the simplex.

And also by analogy, when hij=0, fij(x) can be determined at the vertices of the simplex as the value of the corresponding reference function:

$$f_{ij}(x, y) = f_i(xy). \tag{3}$$

Thus, if we supplement formula (1) with expressions (2) and (3), the function will be defined and continuous on the entire symplicity complex.

The final form of the function, its properties on the interpolation area, in addition to certainty and continuity, completely depend on the type of weight coefficients w(h), q(h) and reference functions fi(x, y).

Bilinear and biquadratic support functions at each point of the interpolation grid (xi, yi) are determined on the basis of a point frame formed from the vertices of the simplexes to which the i-th point belongs.

When using inverse weighting coefficients, the derivative of the function at the boundaries of the simplex suffers a break in the transversal direction, that is, the smoothness of the function depends on the type of weighting coefficients.

It should also be noted that the final form of the interpolation function also depends on the method of constructing the simplicial complex on the point frame. Depending on which vertices are included in the simplex, which reference functions will be used in the calculation, and what the values of the reference coefficients will be. Figure 3 clearly illustrates this dependence.

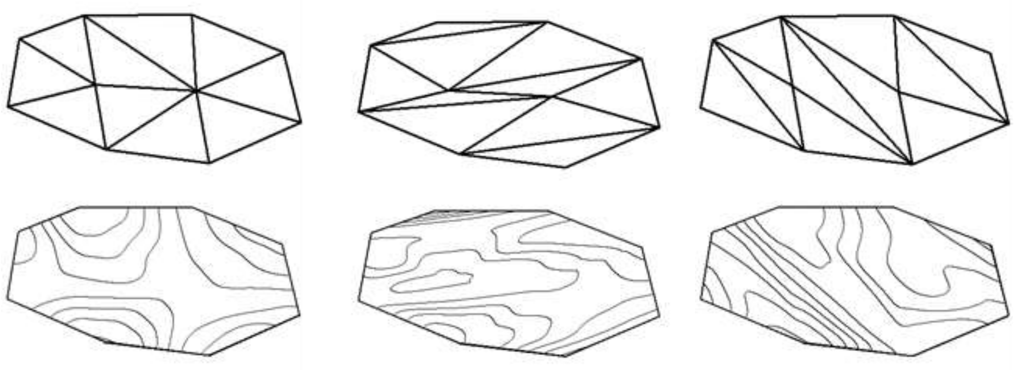

Fig. 3. Dependence of the final result of interpolation on the method of construction of the simplicial complex

The task of modeling the volume of the aquifer over time consists of two stages. At the first stage, on the basis of geospatial data, the upper and lower boundaries of the aquifer are constructed at a certain point in time. Deformation modeling is used to construct the surface: the sea level surface is transformed into the layer boundary surface. The basis is the coordinates of the measurements of the depth of the upper and lower boundary of the layer at sea level, the new basis is the coordinates of the measurements with the corresponding depths.

To use the vector deformation field constructed by simplex weighted interpolation, point basis triangulation is performed. Interpolation is used to calculate the transfer of surface points in the syplicial complex, beyond it simplex weighted extrapolation is used[3].

Modeling of the lower boundary of the aquifer ends here, as this boundary practically does not change over time. The upper boundary of the aquifer, on the other hand, is subject to seasonal fluctuations, so its simulation must be dynamic: the surface must correspond to the target simulation time (ti). For this, the model should be based on data on the current level of groundwater in the wells.

Dynamic modeling of the upper surface of the aquifer constitutes the second stage of deformation modeling of the volume of the aquifer. At this stage, the prototype of the object is the upper surface of the aquifer defined at time t0. The basis is the coordinates of the wells in which the groundwater level is measured, together with the corresponding level at the time t0. The new basis is the coordinates of the same wells, but with the groundwater level determined at the moment of time ti.

In figure 4 shows how the surface is deformed, taking into account measurements of depths at predetermined points (wells).

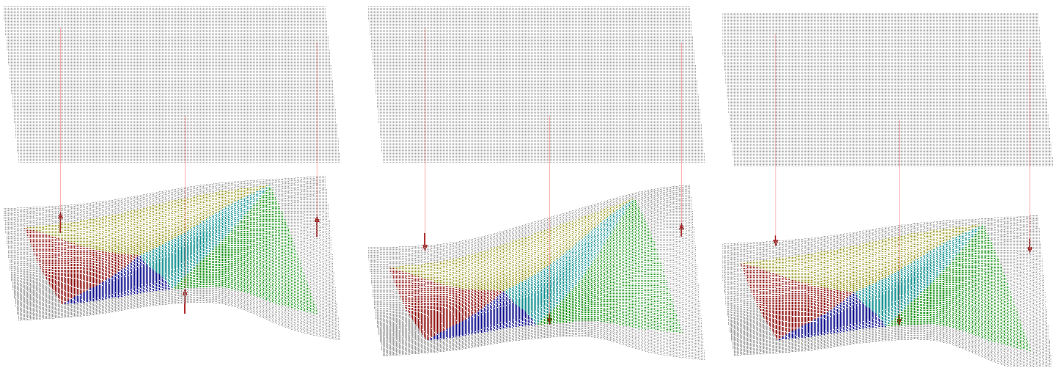

Fig. 4. Modeling of seasonal fluctuation of the upper boundary of the aquifer

The model of the aquifer itself consists of the upper surface, which is modeled regularly, based on data on the current level of groundwater in the wells, and the lower surface of the aquifer, which is modeled once on the basis of georesearch data. The space bounded by these surfaces corresponds to the aquifer model. Figure 5 shows an example of aquifer modeling for three different time points.

Fig. 5. Modeling of changes in the aquifer

Thus, the use of deformation modeling methods, namely, modeling the deformation of geometric objects in a vector field constructed by simplex weighted interpolation, allows obtaining a dynamic model of the aquifer based on georesearch data and current data on the groundwater level.

Conclusions. A method of building a model for tracking changes in the volume of groundwater to determine the relative concentration of radiological emissions is proposed. Further work in this direction will be aimed at improving the interpolation formula by switching to a complex field, which will simplify the model building procedure and reduce data processing time.

Література

1. Сидоренко Ю. В., Каленюк О.С. Симплекса інтерполяція функції двох змінних. *Сучасні проблеми геометричного моделювання* : наукові

нотатки міжнародної науково-практичної конференції, Луцьк: ЛДТУ, 2008. С. 314–319.
2. Сидоренко Ю. В., Каленюк О.С. Симплекса інтерполяція функції багатьох змінних. *Геометричне модулювання та комп'ютерні технології: теорія, практика, освіта*: міжнар. наук.-практ. конф.: наукові нотатки, НТУ «ХПІ», 2009. С. 314–319.
3. Каленюк О. С. Алгоритм екстраполяції для двовимірної симплексної інтерполяції. *Прикладна геометрія та іінженерна графіка*. Київ: КНУБА, 2009. № 81. С. 201–204.
4. Сидоренко Ю. В., Каленюк О.С. Моделювання векторного поля деформації геометричних об'єктів. Dynamical system modelling and stability investigation: міжнар. наук. конф.: тези доповідей, 21–24 трав. 2007. К., 2007. С. 326.
5. Сидоренко Ю.В. Побудова гладких ліній за допомогою параметризованих функцій Гауса. *Прикладна геометрія та інженерна графіка*. К.;КДТУБА, 2001, Вип. 69. С. 63-67.
6. Sydorenko Yu. V., Horodetskyi M.V. Modification of the algorithm for selecting a variable parameter of the Gaussian interpolation function. Control Systems and Computers, 2020, Issue 6 (290), pp. 21-28.

ВІДСТЕЖЕННЯ ЗМІНИ ОБ'ЄМУ ГРУНТОВИХ ВОД ЗА ДОПОМОГОЮ СИМПЛЕКСНОЇ ВАГОВОЇ ІНТЕРПОЛЯЦІЇ

Сидоренко Ю.В., Тарнавський Ю.А., Онисько А.І.

У статті розглянуто представлення об'єкта у векторному полі для проведення розрахунку зміни об'єму ґрунтових вод при визначенні концентрації забруднювачів. В процесі моделювання використовуються методи симплексної вагової інтерполяції.

С того часу як людство почало використовувати атомні електричні станції, виникла необхідність проводити перевірки забруднення довколишнього середовища на наявність радіоактивних забруднень. Особливої актуальності ця задача набула під час аварії на Чорнобильській АЕС. У наш час людство стикається з серйозними викликами в енергетиці, які пов'язані як із природними катаклізмами, такими як повінь, цунамі, землетруси, так і з результатами обстрілів ворога під час повномасштабної війни, яку розв'язала Росія. Багато об'єктів енергетичної інфраструктури знищено ворогом, і велике навантаження зараз лягло на атомні станції. Тому актуальність задачі моніторингу стану довкільна набуває нового сенсу.

Під час проведення перевірки забруднення водоносного шару в районі об'єкта "Укриття" Чорнобильської атомної станції виникла задача відстеження зміни радіоактивного забруднення, враховуючи річну амплітуду *коливання ґрунтових вод. Як з'ясувалось концентрація радіоактивних елементів у воді змінюється при зміні рівня ґрунтових вод. Це відбувається через зменшення об'єму ґрунтових вод, а не як пряв збільшення кількості забруднювачів. Тому виникає необхідність відстеження зміни загального об'єму води у водоносному шарі з одного боку, і її вплив на концентрацію радіоактивних елементів з другого..*

З великої кількості методів, за допомогою яких можна розв'язувати цю задачу, було обрано методи деформаційного моделювання, шляхом представлення об'єкта у векторному полі за допомогою застосування симплексної вагової інтерполяції [4].

Метою досліджень є удосконалення способу визначення зміни об'єму водоносного шару на базі даних георозвідки з урахуванням поточної ситуації по ґрунтових водах. У статті пропонується використання симплексної вагової інтерполяції для побудови векторного поля для отримання відповідної моделі і можливості зорового відстеження процесу зміни рівня ґрунтових вод в режимі реального часу.

Ключові слова: деформаційне моделювання, векторне поле, симплекс, симплексна вагова інтерполяція.

Referenses

1. Sidorenko Yu.V. , Kalenyuk O. S. (2008) Simplex interpolation of a function of two variables. *Suchasni problemi geometrichnogo modelyuvannya*: *naukovi notatki mizhnarodnoyi naukovo-praktichnoyi konferenciyi*. pp. 314–319. Lutsk: LDTU [in Ukrainian].
2. Sidorenko Yu.V. , Kalenyuk O. S. (2009) Simplex interpolation of functions with many changes. *Geometrichne modulyuvannya ta komp'yuterni tehnologiyi: teoriya, praktika, osvita*: mizhnar. nauk.-prakt. konf.: naukovi notatki. pp. 314–319 [in Ukrainian].
3. Kalenyuk O.S. (2009) Extrapolation algorithm for two-dimensional complex interpolation. *Prikladna geometriya ta inzhenerna grafika*, 81. 201–204 [in Ukrainian].
4. Sidorenko Yu.V. , Kalenyuk O.S. (2007) Modeling the vector field of deformation of geometric objects. *Dynamical system modelling and stability investigation*: *mizhnar. nauk. konf.* Kyiv, 326 [in Ukrainian].
5. Sydorenko Iu.V. (2001) Pobudov smooth lines using additional parameterized Gauss functions. *Prykladna geometriya ta inzhenerna grafika*. Kyiv: KDTUBA. 69. 63-67 [in Ukrainian].
6. Sydorenko Yu. V., Horodetskyi M.V. (2020) Modification of the algorithm for selecting a variable parameter of the Gaussian interpolation function. *Control Systems and Computers*, Issue 6 (290), pp. 21-28.