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SOLVING THE RIDDLE OF SPHERICAL PERSPECTIVE OF THE PAINTERS OF THE RENAISSANCE IN NORTHERN EUROPE

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A method has been proposed for constructing spherical perspective using the simplest drawing tools, namely a compass and a ruler. The method is based on the notion that spherical perspective is nothing more than a reflection in a round convex mirror. The clue to the mystery of spherical perspective was the round convex mirror depicted in many portraits in the interior by Jan van Eyck, Robert Campin, Hieronymus Bosch, Pieter Bruegel and other outstanding painters of the Renaissance in Northern Europe. Moreover, it has been suggested that they knew a way to construct a reflection of a geometric figure in a round convex mirror using those drawing tools that were known to painters of the 15th-16th centuries, namely a compass and a ruler, and were able to apply the acquired knowledge to construct the spherical perspective.

Therefore, the proposed method of constructing spherical perspective can be considered as a reconstruction of the geometric constructions with which the artists of the Renaissance in Northern Europe reproduced the visually perceived space on the plane of the picture.

What's more, thanks to the study of the geometry of paintings by the painters of the Renaissance in Northern Europe, it was given the definition of spherical perspective. Let's call spherical perspective the parallel projection of a three-dimensional image of a geometric figure, which is its reflection relative to the sphere, onto a plane tangent to it.

Thus, in this work we have given an outline of the theory of spherical perspective that corresponds to the features of natural human visual perception. In addition to its theoretical value, our research also has practical significance, which consists in the fact that spherical perspective, represented as a reflection in a spherical mirror, can be applied in 'virtual reality' technology. This will allow the picture of a three-dimensional scene to be brought closer to natural visual perception and to convey objects in its foreground without the monstrous distortions inherent in linear perspective.

Key words: spherical perspective, geometric constructions using a compass and ruler, art of the Renaissance in Northern Europe.

Formulation of the problem. In the visual arts, spherical perspective is understood as the central projection of points of three-dimensional space onto the inner surface of a sphere, in which its center coincides with the center of

projection. Despite the fact that many authors of works on the theory of perspective pointed out that spherical perspective is similar to reflection in a spherical mirror, none of them had the notion that spherical perspective is a reflection in a spherical mirror. Unfortunately, instead of making out spherical perspective as a reflection relative to a sphere, they make out perspectives in a circle with four, five, and even six points of convergence, which are nothing more than falsifications of reflection in a round convex mirror [1–3].

A clue to the mystery of spherical perspective is the round convex mirror depicted in many portraits in the interior by the painters of the Renaissance in Northern Europe. Perhaps the first time a round convex mirror appears in the famous painting by the Dutch painter Jan van Eyck (1385–1441) 'The Portrait of the Arnolfini Couple' (1434, London, National Gallery). The mirror hangs on the far wall of the narrow room, in a round frame decorated with ten miniatures arranged in a circle with scenes from the life of Jesus Christ. In the depths of the mirror, in the doorway, we see a portrait of the painter himself, dressed in a blue caftan and a long fur-lined cloak with slits for the arms, called a pelisson. Perhaps Jan van Eyck deliberately introduced a round convex mirror into the painting to show himself, but the scene depicted in the mirror is conveyed with a subtle knowledge of the optical laws of reflection in a spherical mirror. The next painter to show a round convex mirror was the Dutch painter Robert Campin (1378–1444), who depicted the scene 'John the Baptist and the Franciscan Heinrich von Werl' on the left panel of the Weil Altarpiece (1438, Madrid, Prado Museum). The mirror hangs on a wooden screen pushed into the far corner of a narrow room, and with unsurpassed accuracy reproduces her reflection in the convex mirror surface. Moreover, a spherical mirror is introduced into the scene 'Pride', included in the painting 'The Seven Deadly Sins' (1480–1490, Madrid, Prado Museum) by the Dutch painter Hieronymus Bosch (1450–1516). If you look closely at the painting, in the depths of the interior we will see a mirror held by the Devil in front of a woman proud of her beauty and the richness of her attire. What's more, her depravity is pointed out by the apple placed on the windowsill, hinting at the fruit of the Tree of Knowledge, which the insidious temptress Eve treated the kind and simple-minded Adam [1].

Moreover, we assume that Jan van Eyck, Robert Campin, Hieronymus Bosch and Pieter Bruegel all knew how to construct a reflection of a geometric figure in a round convex mirror using those drawing tools that were known to painters of the 15th-16th centuries, namely a compass and a ruler, and were able to apply the acquired knowledge to construct the pictorial space in their paintings.

Indeed, in the painting by Pieter Bruegel (1525–1569) 'The Tower of Babel' (1563, Vienna, Kunsthistorisches Museum), the lines of the outline of the tower clearly curve and seem to converge at one point taken on a sphere, but at the same time its image does not fall out of the picture space and forms a single whole with it. In our opinion, this is explained by the fact that Pieter Bruegel

possessed knowledge of spherical perspective and knew how to use it to construct a pictorial space that possessed continuity and homogeneity, as well as linking together figures and the spatial gaps between them.

Analysis of recent research and publications. Although many authors of works on perspective theory have pointed out that spherical perspective is similar to reflection in a spherical mirror, none of them had the notion that spherical perspective is reflection in a spherical mirror. Unfortunately, instead of working out spherical perspective as reflection relative to a sphere, they worked out perspectives in a circle with four, five and even six points of convergence, which are nothing more than falsifications of reflection in a round convex mirror [2, 3].

At the same time, there is a work in which spherical perspective is considered as a reflection on a sphere or cylinder of rotation [4]. In it, the reflection of a point of space is understood as a point on a sphere or cylinder of revolution at which the angle between the ray emanating from the point in space and the normal to the surface is equal to the angle between the ray passing through the point of view and the same normal to the surface. This notion of reflection on any surface contradicts its definition, according to which the reflection of a point of space is a point of space that is symmetrical to it relative to the point of intersection of the mirror surface with a ray perpendicular to it. Then the point that a person observes is the point of intersection with the surface of the mirror of the ray connecting the reflection with the point of view from which the observation is being made. It follows that the spherical perspective is understood as the central projection of the reflection of a point of space from a sphere or cylinder onto their surface or plane, tangent to them and perpendicular to the line connecting the point of view with the center of the sphere or a point on the axis of the cylinder.

Moreover, the method of constructing spherical perspective proposed in the work under consideration involves constructing catacaustics of rays reflected from both the inner and outer surfaces of a sphere or cylinder, and two branches of a hyperbola. What's more, the solution to the problem of constructing the intersection points of a hyperbola with a circle from which a ray of light is reflected requires finding the roots of an algebraic equation of the fourth order. Consequently, this problem cannot be solved by a compass and ruler. Therefore, the method of constructing a reflection on a sphere or cylinder of revolution, proposed in the work under consideration, could not have been known to the painters of the Renaissance in Northern Europe.

Formulating the purposes of the article. Thus, the aim of our study is to reconstruct spherical perspective, the foundations of which were laid by the painters of the Renaissance in Northern Europe in the 15th-16th centuries, and its application to the conveyance of visually perceived space on the plane of the picture.

Main part. Let's show that spherical perspective can be constructed using a compass and a ruler. This is necessary to prove that the level of mathematical

knowledge in the 15th-16th centuries allowed painters and mathematicians of the Renaissance in Northern Europe to work out a theory of spherical perspective and construct it using the simplest drawing tools.

Let's show in Fig. 1, a the construction of a linear perspective of a geometric figure resembling a simple house of an honest man, 6800 mm wide, 5500 mm deep and 5550 mm high. What's more, the construction was carried out using the 'architects' way'.

Let's show in Fig. 1, b the construction of a spherical perspective of the same geometric figure by means of its reflection in a round convex mirror, the diameter of which is equal to 13245 mm.

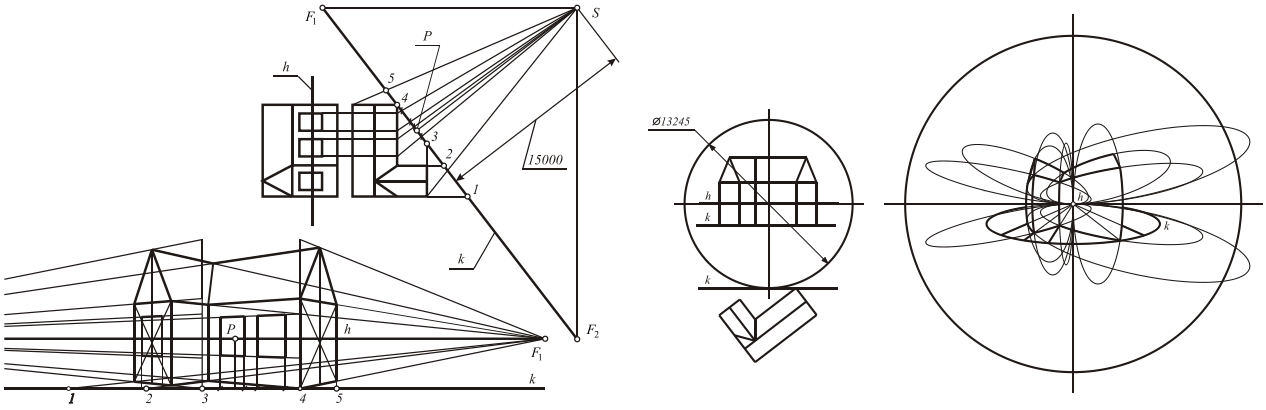

Fig. 1. Linear and spherical perspectives of the same geometric figure

Since in Fig. 1, b spherical perspective is represented as a reflection in a round convex mirror, we consider the proposed method of constructing spherical perspective as a reconstruction of geometric constructions with which the painters of the Renaissance in Northern Europe reproduced visually perceived space on the plane of the picture.

What's more, thanks to the study of the geometry of paintings by the painters of the Renaissance in Northern Europe, we can give the definition of spherical perspective. Let's call spherical perspective the parallel projection of a three-dimensional image of a geometric figure, which is its reflection relative to the sphere, onto a plane tangent to it.

Let's take a look at the differences that exist between spherical and linear perspectives:

– if in linear perspective the horizon is depicted by a straight line, then in spherical perspective the horizon is depicted by a point coinciding with the center of the sphere. This is explained by the fact that in spherical perspective the point of convergence of parallel lines drawn in the object plane at some angle to the basis of the picture plane is the center of the sphere;

– if in linear perspective the image of a point of space is located at the intersection of straight lines, which are pictorial traces of two projecting planes, in spherical perspective the image of a point of space is located at the

intersection of curves, which are spherical perspectives of two straight lines belonging to a geometric figure. This confirms that in spherical perspective, unlike linear perspective, there are no concepts of either projecting planes or their subject and pictorial traces.

It is remarkable that the optical illusion caused by the reflection of space in the convex surface of a spherical mirror can be observed in such early paintings by Hieronymus Bosch as ‘Operation Folly’ (1490, Prado, Madrid), ‘The Seven Deadly Sins’ (1480–1490, Madrid, Prado) and ‘The Ship of Fools’ (1490–1495, Paris, Louvre). Additionally, the notion of the spherical shape of the earth’s space continues to be present in many of Hieronymus Bosch’s later works, for example, in the central panel of the Vienna altarpiece ‘The Last Judgment’ (1505-1510, Vienna, Academy of Arts), in the central panel of the triptych ‘The Haywain’ (1490-1495, Madrid, Prado) and in the middle panel of the triptych ‘The Garden of Earthly Delights’ (1500-1505, Madrid, Prado).

Hieronymus Bosch’s cosmographic view of world space was shared by the German painter Albrecht Altdorfer (1480–1538), who reflected it in the painting ‘The Battle of Alexander the Great with the Persian King Darius’ (1529, Munich, Alte Pinakothek), which is considered the greatest insight into the essence of landscape ever achieved by painters of previous and future generations.

The notion of world space, which is both boundless and at the same time closed within a sphere, was the basis for Pieter Bruegel’s cosmic landscapes such as ‘The Fall of Icarus’ (1555–1558, Brussels, Royal Museum of Fine Arts), ‘The Battle of Shrovetide and Lent’ (1559, Vienna, Kunsthistorisches Museum) and ‘Hunters in the Snow’ (1565, Vienna, Kunsthistorisches Museum).

Let’s take another look at Pieter Bruegel’s (1525–1569) painting ‘The Tower of Babel’ (1563, Vienna, Kunsthistorisches Museum) and carefully study its geometry. The first thing we notice is that the lines outlining the tower are curved in such a way that they appear concave to the viewer. However, if we look into a round convex mirror, the straight lines will be reflected in it as curves intersecting at a point in the depth of the picture, that is, they will appear to us as curved. It follows that the walls of the tower are segments of curves intersecting at the center of a spherical mirror. What’s more, the center of the spherical mirror is not in front of us, as it would be if we were looking into it, but is located behind us. This means that we are not standing in front of a round convex mirror, but are inside it and see in front of us the reflection of objects lying on the other side of the spherical surface.

We believe that the painting ‘The Tower of Babel’ is an example of medieval notions that the earthly world is nothing more than a reflection of the divine world in a spherical mirror, which the painters of the Renaissance in Northern Europe understood as the firmament. Therefore, Pieter Bruegel’s painting ‘The Tower of Babel’ not only reflects the system of spatial constructions adopted as a model in the 15th–16th centuries by the painters of the Renaissance in Northern Europe, but also reflects their worldview.

Consequently, we have given visual proof of the assertion that there are no obstacles that could prevent the painters of the Renaissance in Northern Europe from constructing spherical perspective using the simplest drawing instruments, namely a compass and a ruler.

Conclusions. Thus, in this work we have given an outline of the theory of spherical perspective that corresponds to the features of natural human visual perception. In addition to its theoretical value, our research also has practical significance, which consists in the fact that spherical perspective, represented as a reflection in a spherical mirror, can be applied in 'virtual reality' technology [5]. This will allow the picture of a three-dimensional scene to be brought closer to natural visual perception and to convey objects in its foreground without the monstrous distortions inherent in linear perspective.

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РОЗГАДКА ТАЄМНИЦІ СФЕРИЧНОЇ ПЕРСПЕКТИВИ ХУДОЖНИКІВ ПІВНІЧНОГО ВІДРОДЖЕННЯ

Ніцин Олександр

Запропоновано спосіб побудови сферичної перспективи за допомогою найпростіших креслярських інструментів, а саме: циркуля та лінійки. Спосіб заснований на уявленні про те, що сферична перспектива є не що інше, як віддзеркалення у круглому опуклому дзеркалі. Підказкою до розкриття таємниці сферичної перспективи послужило кругле опукле дзеркало, яке зображується на багатьох портретах в інтер'єрі Яна ван Ейка, Робера Кампена, Ієроніма Босха, Пітера Брейгеля та інших видатних художників Північного Відродження. Більш того, було припущено, що вони знали спосіб, за допомогою якого можна побудувати

відображення геометричної фігури в круглому опуклому дзеркалі за допомогою циркуля та лінійки, та вміли застосовувати отримані знання для побудови простору у своїх мальовничих полотнах.

Тому запропонований спосіб побудови сферичної перспективи можна розглядати як реконструкцію геометричних побудов, за допомогою яких художники Північного Відродження відтворювали на площині картини простір, що візуально сприймається.

Причому завдяки уважному прочитанню геометрії картин художників Північного Відродження було дано визначення сферичної перспективи. Будемо називати сферичною перспективою паралельну проекцію тривимірного образу геометричної фігури, що є її віддзеркаленням щодо сфери, на площину, що торкається до неї.

Таким чином, у цій роботі було дано нариси теорії сферичної перспективи, що відповідає особливостям природного зорового сприйняття людини. Крім теоретичної цінності, у нашого дослідження є і практичне значення, яке полягає в тому, що сферичну перспективу, представлену як відображення у сферичному дзеркалі, можна застосувати у технології «віртуальної реальності». Це дозволить наблизити картину тривимірної сцени до природного зорового сприйняття людини і передати предмети на її передньому плані без суттєвих спотворень, притаманних лінійній перспективі.

Ключові слова: сферична перспектива, геометричні побудови за допомогою циркуля та лінійки, мистецтво Північного Відродження.

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